Chapter (8) Circular Measure

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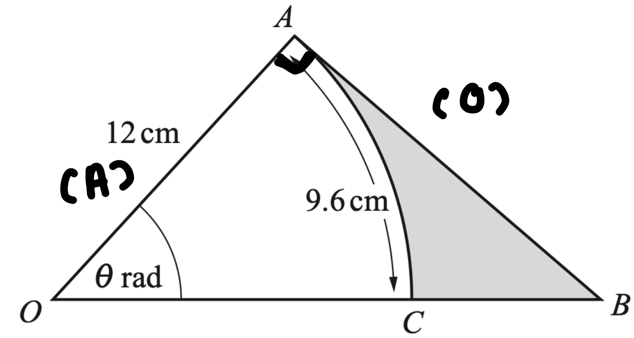
1.



The diagram shows the right-angled triangle *OAB.*The point *C* lies on the line *OB.* Angle *OAB*= $\frac{\pi}{2}$ radians and angle *AOB* = θ radians. *AC* is an arc of the circle, centre *O*, radius 12 cm and *AC* has length 9.6 cm.

a. Find the value of θ.

[2]

$9.6 = r\theta$

$\theta = \frac{9.6}{12} = 0.8$ rad

b. Find the area of the shaded region.

[4]

Area of sector $= \frac{1}{2} r^2\theta$

$= \frac{1}{2} 12^2 \times 0.8$

$= 57.6$ cm^2

$\tan 0.8 = \frac{x}{12}$

$x = 12.36$

Area of Δ $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 12 \times 12.36$

$= 74.16$

shaded Area $= 74.16 - 57.6$

$= 16.56$ cm^2

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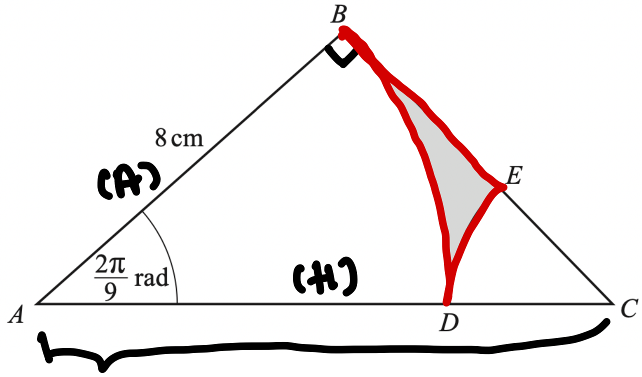
2.



The diagram shows a right-angled triangle ABC with $AB = 8$ cm and angle $ABC = \frac{\pi}{2}$ radians. The points D and E lie on AC and BC respectively. BAD and ECD are sectors of the circles with centres A and C respectively. Angle $BAD = \frac{2\pi}{9}$ radians.

a. Find the area of the shaded region [6]

Area of sector $BAD = \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 8^2 \times \frac{2\pi}{9} = \frac{64}{9}\pi \text{ cm}^2$

$\cos\frac{2\pi}{9} = \frac{8}{x}$

$x = 10.44$

$CD = 10.44 - 8 = 2.44$

Area of sector $DCE = \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 2.44^2 \times \left(\frac{\pi}{2} - \frac{2\pi}{9}\right)$

$= \frac{1}{2} \times 2.44^2 \times \frac{5\pi}{18}$

$= 2.598 \text{ cm}^2$

Area of $\Delta = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 8 \times 10.44 \times \sin\frac{2\pi}{9}$

$= 26.84$

shaded $= 26.84 - \left(2.598 + \frac{64\pi}{9}\right) = 1.90 \text{ cm}^2$

b. Find the perimeter of the shaded region. [3]

Arc length $BD = r\theta$

$= 8 \times \frac{2\pi}{9} = \frac{16\pi}{9}$

Arc length $ED = r\theta$

$= 2.44 \times \frac{5\pi}{18}$

$= \frac{61\pi}{90}$

$BC^2 = 10.44^2 - 8^2$

$BC = 6.708$ cm

$BE = 6.708 - 2.44$

$= 4.27$

shaded Perimeter $= \frac{16\pi}{9} + \frac{61\pi}{90} + 4.27$

$= 11.98$ cm

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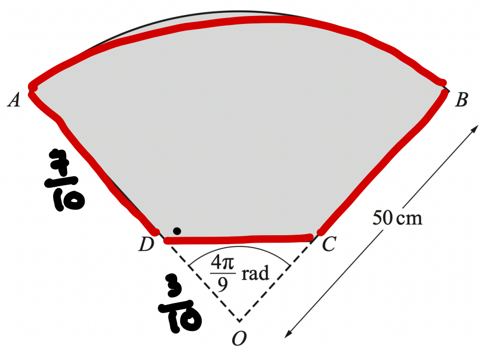
3.



The diagram shows a company logo, *ABCD*. The logo is part of a sector, *AOB*, of a circle, centre *O* and radius 50 cm. The points *C* and *D* lie on *OB* and *OA* respectively. The lengths *AD* and *BC* are equal and *AD* : *AO* is 7 : 10. The angle *AOB* is $\frac{4\pi}{9}$ radians.

a. Find the perimeter of *ABCD*. [5]

Arc $AB = r\theta = 50 \times \frac{4\pi}{9} = \frac{200\pi}{9}$ cm

$AD = \frac{7}{10} \times 50$

$= 35$ cm

$BC = 35$ cm

$OD = 50 - 35$

$= 15$

$CD^2 = 15^2 + 15^2 - 2 \times 15 \times 15 \times \cos \frac{4\pi}{9}$

$CD = 19.28$

Shaded Perimeter $= 19.28 + 70 + \frac{200\pi}{9}$

$= 159$ cm

b. Find the area of *ABCD*. [3]

Area of sector $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 50^2 \times \frac{4\pi}{9}$

$= \frac{5000}{9} \pi$

Area of $\Delta = \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 15 \times 15 \times \sin \frac{4\pi}{9}$

$= 110.79$

shaded Area $= \frac{5000}{9} \pi - 110.79$

$= 1634.5 \text{ cm}^2$

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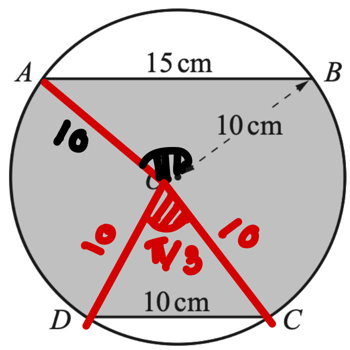
4.



The diagram shows a circle with centre *O* and radius 10 cm. The points *A*, *B*, *C* and *D* lie on the circle such that the chord *AB* = 15 cm and the chord *CD* = 10 cm. The chord *AB* is parallel to the chord *DC*.

a. Show that the angle *AOB* is 1.70 radians correct to 2 decimal places. [2]

$15^2 = 10^2 + 10^2 - 2 \times 10 \times 10 \times \cos\theta$

$225 - 200 = -200\cos\theta$

$\cos\theta = \frac{25}{-200}$

$\theta = 1.696 \approx 1.70$ (shown)

b. Find the perimeter of the shaded region. [4]

$\angle DOC = \frac{\pi}{3}$

$\angle AOD = (2\pi - \frac{\pi}{3} - 1.70) \div 2$

$= 1.768$

Arc $AD = r\theta$

$= 17.68$

shaded perimeter $= 17.68 \times 2 + 10 + 15$

$= 28.5$ cm

60.4 cm

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c. Find the area of the shaded region. [4]

Area of sector $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 100 \times 1.768$

$= 88.4$

Area of $\Delta = \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 100 \times \sin \frac{\pi}{3}$

$= 43.3$

Area of $\Delta = \frac{1}{2} \times 100 \times \sin 1.70$

$= 49.6$

shaded Area $= 88.4 \times 2 + 43.3 + 49.6$

$= 270 \text{ cm}^2$

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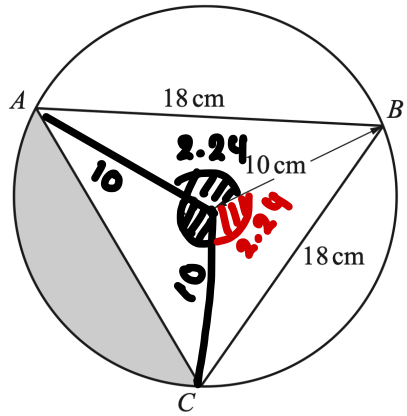
5.



The diagram shows a circle centre *O*, radius 10 cm. The points *A*, *B* and *C* lie on the circumference of the circle such that *AB* = *BC* = 18 cm.

a. Show that angle *AOB* = 2.24 radians correct to 2 decimal places. [3]

$\sin\theta = \frac{9}{10}$

$\theta = 1.1198$

$\angle AOB = 2\theta = 2.2395$

≈ 2.24 (shown)

b. Find the perimeter of the shaded region. [5]

$\angle COB = 2.24$

$\angle AOC = 2\pi - 2.24 - 2.24$

$= 1.803$

$AC^2 = 10^2 + 10^2 - 2\times 10\times 10 \times \cos 1.803$

$AC = 15.69$ cm

Arc $AC = r\theta$

$= 10 \times 1.803$

$= 18.03$

shaded Perimeter $= 15.69 + 18.03$

$= 33.7$ cm

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c. Find the area of the shaded region.

[3]

Area of sector $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 100 \times 1.803$

$= 90.15 \text{ cm}^2$

Area of $\Delta = \frac{1}{2} ab\sin C$

$= \frac{1}{2} \times 100 \times \sin 1.803$

$= 48.658 \text{ cm}^2$

shaded Area $= 90.15 - 48.658$

$= 41.5 \text{ cm}^2$

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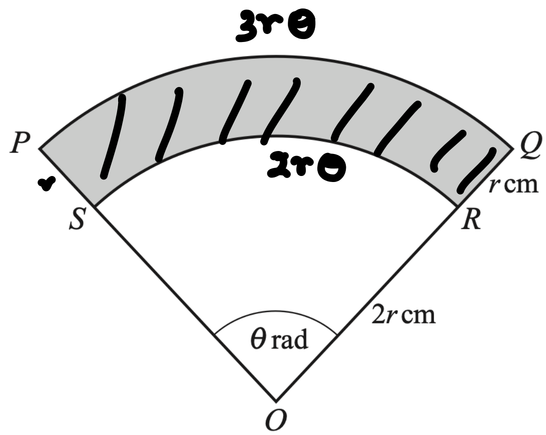
6



The diagram shows a sector *OPQ* of the circle centre *O*, radius 3*r* cm. The points *S* and *R* lie on *OP* and *OQ* respectively such that *ORS* is a sector of the circle centre *O*, radius 2*r* cm. The angle *POQ* = θ radians. The perimeter of the shaded region *PQRS* is 100 cm.

a. Find θ in terms of *r*. [2]

$$100 = r + r + 2r\theta + 3r\theta$$

$$100 = 2r + 5r\theta$$

$$5r\theta = 100 - 2r$$

$$\theta = \frac{100 - 2r}{5r}$$

b. Hence show that the area, $A\ cm^2$, of the shaded region *PQRS* is given by $A = 50r - r^2$. [2]

Area of sector POQ $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times (3r)^2 \times \frac{100 - 2r}{5r}$

$= \frac{1}{\cancel{2}} \times 9r^{\cancel{2}} \times \frac{\cancel{100 - 2r}^{\,50 - r}}{5\cancel{r}}$

$= \frac{9r}{5}(50 - r) = 90r - \frac{9r^2}{5}$

Area of sector SOR $= \frac{1}{2} r^2 \theta$

$= \frac{1}{\cancel{2}} \times 4r^{\cancel{2}} \times \frac{\cancel{100 - 2r}^{\,50 - r}}{5\cancel{r}}$

$= \frac{4r}{5}(50 - r) = 40r - \frac{4r^2}{5}$

shaded Area $= 90r - \frac{9r^2}{5} - 40r + \frac{4r^2}{5} = 50r - \frac{5r^2}{5}$

$= 50r - r^2$ (shown)

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